CHAPTER 7
QUANTUM THEORY AND THE ELECTRONIC STRUCTURE OF ATOMS

PROBLEM-SOLVING STRATEGIES AND TUTORIAL SOLUTIONS

TYPES OF PROBLEMS

Problem Type 1: Calculating the Frequency and Wavelength of an Electromagnetic Wave.

Problem Type 2: Calculating the Energy of a Photon.

Problem Type 3: Calculating the Energy, Wavelength, or Frequency in the Emission Spectrum of a Hydrogen Atom.

Problem Type 4: The de Broglie Equation: Calculating the Wavelengths of Particles.

Problem Type 5: Quantum Numbers.
- **(a)** Labeling an atomic orbital.
- **(b)** Counting the number of orbitals associated with a principal quantum number.
- **(c)** Assigning quantum numbers to an electron.
- **(d)** Counting the number of electrons in a principal level.

Problem Type 6: Writing Electron Configurations and Orbital Diagrams.

PROBLEM TYPE 1: CALCULATING THE FREQUENCY AND WAVELENGTH OF AN ELECTROMAGNETIC WAVE

All types of *electromagnetic radiation* move through a vacuum at a speed of about 3.00×10^8 m/s, which is called the speed of light (c). Speed is an important property of a wave traveling through space and is equal to the product of the wavelength and the frequency of the wave. For electromagnetic waves

$$c = \lambda\nu \qquad (7.1)$$

Equation (7.1) can be rearranged as necessary to solve for either the wavelength (λ) or the frequency (ν).

EXAMPLE 7.1

A certain AM radio station broadcasts at a frequency of 6.00×10^2 kHz. What is the wavelength of these radio waves in meters?

Strategy: We are given the frequency of an electromagnetic wave and asked to calculate the wavelength. Rearranging Equation (7.1) gives:

$$c = \lambda\nu$$

$$\lambda = \frac{c}{\nu}$$

Solution: Because the speed of light has units of m/s, we must convert the frequency from units of kHz to Hz (s^{-1})

$$(6.00 \times 10^2 \text{ kHz}) \times \frac{1000 \text{ Hz}}{1 \text{ kHz}} = 6.00 \times 10^5 \text{ Hz} = 6.00 \times 10^5 \text{ s}^{-1}$$

Substituting in the frequency and the speed of light constant, the wavelength is:

$$\lambda = \frac{c}{\nu} = \frac{3.00 \times 10^8 \frac{\text{m}}{\text{s}}}{6.00 \times 10^5 \frac{1}{\text{s}}} = \mathbf{5.00 \times 10^2 \text{ m}}$$

Check: Look at Figure 7.4 of the text to confirm that this wavelength corresponds to a radio wave.

EXAMPLE 7.2
What is the frequency of light that has a wavelength of 665 nm?

Strategy: We are given the wavelength of an electromagnetic wave and asked to calculate the frequency. Rearranging Equation (7.1):

$$c = \lambda\nu$$

$$\nu = \frac{c}{\lambda}$$

Solution: Since the speed of light has units of m/s, we must convert the wavelength from units of nm to m.

$$665 \text{ nm} \times \frac{1 \times 10^{-9} \text{ m}}{1 \text{ nm}} = 6.65 \times 10^{-7} \text{ m}$$

Substituting in the wavelength and the speed of light (3.00×10^8 m/s), the frequency is:

$$\nu = \frac{c}{\lambda} = \frac{3.00 \times 10^8 \frac{\text{m}}{\text{s}}}{6.65 \times 10^{-7} \text{ m}} = \mathbf{4.51 \times 10^{14} \text{ s}^{-1} = 4.51 \times 10^{14} \text{ Hz}}$$

PRACTICE EXERCISE

1. Domestic microwave ovens generate microwaves with a frequency of 2.450 GHz. What is the wavelength of this microwave radiation?

Text Problems: 7.8, 7.12, **7.16**

PROBLEM TYPE 2: CALCULATING THE ENERGY OF A PHOTON

Max Planck said that atoms and molecules could emit (or absorb) energy only in discrete quantities. Planck gave the name *quantum* to the smallest quantity of energy that can be emitted (or absorbed) in the form of electromagnetic radiation. The energy E of a single quantum of energy is given by

$$E = h\nu \qquad \text{(7.2, text)}$$

where,

h is Planck's constant = 6.63×10^{-34} J·s
ν is the frequency of radiation

EXAMPLE 7.3

The yellow light given off by a sodium vapor lamp has a wavelength of 589 nm. What is the energy of a single photon of this radiation?

Strategy: We are given the wavelength of an electromagnetic wave and asked to calculate its energy. Equation (7.2) of the text relates the energy and frequency of an electromagnetic wave.

$$E = h\nu$$

The relationship between frequency and wavelength is:

$$\nu = \frac{c}{\lambda}$$

Substituting for the frequency gives,

$$E = \frac{hc}{\lambda}$$

Solution: Because the speed of light is in units of m/s, we must convert the wavelength from units of nm to m.

$$589 \cancel{\text{nm}} \times \frac{1 \times 10^{-9}\text{ m}}{1 \cancel{\text{nm}}} = 5.89 \times 10^{-7}\text{ m}$$

Substituting in Planck's constant, the speed of light constant, and the wavelength, the energy is:

$$E = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34}\text{ J}\cdot\cancel{\text{s}})\left(3.00 \times 10^{8}\frac{\cancel{\text{m}}}{\cancel{\text{s}}}\right)}{5.89 \times 10^{-7}\cancel{\text{m}}} = \mathbf{3.38 \times 10^{-19}\ J}$$

Check: We expect the energy of a single photon to be a very small energy as calculated above, 3.38×10^{-19} J.

PRACTICE EXERCISE

2. The red line in the spectrum of lithium occurs at 670.8 nm. What is the energy of a photon of this light? What is the energy of 1 mole of these photons?

3. The light-sensitive compound in most photographic films is silver bromide (AgBr). When the film is exposed, assume that the light energy absorbed dissociates the molecule into atoms. (The actual process is more complex.) If the energy of dissociation of AgBr is 1.00×10^{2} kJ/mol, find the wavelength of light that is just able to dissociate AgBr.

Text Problems: 7.16, 7.18, 7.20

PROBLEM TYPE 3: CALCULATING THE ENERGY, WAVELENGTH, OR FREQUENCY IN THE EMISSION SPECTRUM OF A HYDROGEN ATOM

Using arguments based on electrostatic interaction and Newton's laws of motion, Neils Bohr showed that the energies that the electron in the hydrogen atom can possess are given by:

$$E_n = -R_H\left(\frac{1}{n^2}\right) \qquad \text{(7.5, text)}$$

where,

R_H is the Rydberg constant = 2.18×10^{-18} J

n is the principal quantum number that has integer values

During the emission process in a hydrogen atom, an electron initially in an excited state characterized by the principal quantum number n_i drops to a lower energy state characterized by the principal quantum number n_f. This lower energy state may be either another excited state or the ground state. The difference between the energies of the initial and final states is:

$$\Delta E = E_f - E_i$$

Substituting Equation (7.5) of the text into the above equation gives:

$$\Delta E = \left(\frac{-R_H}{n_f^2}\right) - \left(\frac{-R_H}{n_i^2}\right)$$

$$\Delta E = R_H\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)$$

Furthermore, since this transition results in the emission of a photon of frequency ν and energy *h*ν (See Problem Type 2), we can write:

$$\Delta E = h\nu = R_H\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right) \qquad \text{(7.6, text)}$$

EXAMPLE 7.4

What wavelength of radiation will be emitted during an electron transition from the $n = 5$ state to the $n = 1$ state in the hydrogen atom? What region of the electromagnetic spectrum does this wavelength correspond to?

Strategy: We are given the initial and final states in the emission process. We can calculate the energy of the emitted photon using Equation (7.6) of the text. Then, from this energy, we can solve for the wavelength. The value of Rydberg's constant is 2.18×10^{-18} J.

Solution: From Equation (7.6) we write:

$$\Delta E = R_H\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)$$

$$\Delta E = (2.18 \times 10^{-18}\ \text{J})\left(\frac{1}{5^2} - \frac{1}{1^2}\right)$$

$$\Delta E = -2.09 \times 10^{-18}\ \text{J}$$

The negative sign for ΔE indicates that this is energy associated with an emission process. To calculate the wavelength, we will omit the minus sign for ΔE because the wavelength of the photon must be positive. We know that

$$\Delta E = h\nu$$

We also know that $\nu = \frac{c}{\lambda}$. Substituting into the above equation gives:

$$\Delta E = \frac{hc}{\lambda}$$

Solving the equation algebraically for the wavelength, then substituting in the known values gives:

$$\lambda = \frac{hc}{\Delta E} = \frac{(6.63 \times 10^{-34}\ \cancel{\text{J}}\cdot\cancel{\text{s}})\left(3.00 \times 10^{8}\ \frac{\text{m}}{\cancel{\text{s}}}\right)}{2.09 \times 10^{-18}\ \cancel{\text{J}}} = \mathbf{9.52 \times 10^{-8}\ m}$$

To determine the region of the electromagnetic spectrum that this wavelength corresponds to, we should convert the wavelength from units of meters to nanometers, and then compare the value to Figure 7.4 of the text.

$$(9.52 \times 10^{-8} \cancel{m}) \times \frac{1 \text{ nm}}{1 \times 10^{-9} \cancel{m}} = 95.2 \text{ nm}$$

Checking Figure 7.4, we see that the ultraviolet region of the spectrum is centered at a wavelength of 10 nm. Therefore, this emission is in the ultraviolet (UV) region of the electromagnetic spectrum.

PRACTICE EXERCISE

4. A hydrogen emission line in the ultraviolet region of the spectrum at 95.2 nm corresponds to a transition from a higher energy level n_i to the $n = 1$ level. What is the value of n_i for the higher energy level?

Text Problems: 7.30, **7.32**, 7.34

PROBLEM TYPE 4: THE DE BROGLIE EQUATION: CALCULATING THE WAVELENGTHS OF PARTICLES

Albert Einstein showed that light (electromagnetic radiation) can possess particle like properties. De Broglie reasoned that if waves can behave like particles, then particles can exhibit wave properties. De Broglie deduced that the particle and wave properties are related by the expression:

$$\lambda = \frac{h}{mu} \qquad \text{(7.8, text)}$$

where,

λ is the wavelength associated with the moving particle
m is the mass of the particle
u is the velocity of the particle

Equation (7.8) of the text implies that a particle in motion can be treated as a wave, and a wave can exhibit the properties of a particle.

EXAMPLE 7.5

When an atom of Th-232 undergoes radioactive decay, an alpha particle, which has a mass of 4.0 amu, is ejected from the Th nucleus with a velocity of 1.4 × 10^7 m/s. What is the de Broglie wavelength of the alpha particle?

Strategy: We are given the mass and the velocity of the alpha particle and asked to calculate the wavelength. We need the de Broglie equation, which is Equation (7.8) of the text. Note that because the units of Planck's constant are J·s, m must be in kg and u must be in m/s (1 J = 1 kg·m^2/s^2).

Solution: Because Planck's constant has units of J·s, and 1 J = 1 kg·m^2/s^2, the mass of the alpha particle must be expressed in kilograms. Since one particle has a mass of 4.0 amu, a mole of alpha particles will have a mass of 4.0 g. A reasonable strategy to complete the conversion is:

g/mol → kg/mol → kg/particle

$$\frac{4.0 \cancel{g}}{1 \cancel{mol}} \times \frac{1 \text{ kg}}{1000 \cancel{g}} \times \frac{1 \cancel{mol}}{6.022 \times 10^{23} \text{ particles}} = 6.6 \times 10^{-27} \text{ kg/particle}$$

Substitute the known quantities into Equation (7.8) to solve for the wavelength.

$$\lambda = \frac{h}{mu} = \frac{(6.63 \times 10^{-34}\ \cancel{J}\cdot\cancel{s}) \times \left(\frac{1\ \cancel{kg}\cdot m^{\cancel{2}}/\cancel{s^2}}{1\ \cancel{J}}\right)}{(6.6 \times 10^{-27}\ \cancel{kg})\left(1.4 \times 10^7\ \frac{\cancel{m}}{\cancel{s}}\right)} = \mathbf{7.2 \times 10^{-15}\ m}$$

The wavelength is smaller than the diameter of the thorium nucleus, which is about 2×10^{-14} m. Is this what you would expect?

PRACTICE EXERCISE

5. The average kinetic energy of a neutron at 25°C is 6.2×10^{-21} J. What is the de Broglie wavelength of an average neutron? The mass of a neutron is 1.008 amu. (**Hint:** kinetic energy = $\frac{1}{2}mu^2$)

Text Problems: 7.40, 7.42

PROBLEM TYPE 5: QUANTUM NUMBERS

See Section 7.6 of your text for a complete discussion. In quantum mechanics, three quantum numbers are required to describe the distribution of electrons in atoms.

(1) **The principal quantum number (n).** In a hydrogen atom, the value of n determines the energy of an orbital. It can have integral values 1, 2, 3, and so forth.

(2) **The angular momentum quantum number (l)** tells us the "shape" of the orbitals. l has possible integral values from 0 to $(n - 1)$. The value of l is generally designated by the letters s, p, d, ..., as follows:

l	0	1	2	3	4	5
Name of orbital	s	p	d	f	g	h

(3) **The magnetic quantum number (m_l)** describes the orientation of the orbital in space. For a certain value of l, there are $(2l + 1)$ integral values of m_l, as follows:

$$-l, (-l + 1), \ldots 0, \ldots (+l - 1), +l$$

The number of m_l values indicates the number of orbitals in a subshell with a particular l value.

Finally, there is a fourth quantum number that tells us the spin of the electron. The **electron spin quantum number (m_s)** has values of +1/2 or −1/2, which correspond to the two spinning motions of the electron.

A. Labeling an atomic orbital

Strategy: To "label" an atomic orbital, you need to specify the three quantum numbers (n, l, m_l) that give information about the distribution of electrons in orbitals. Remember, m_s tells us the spin of the electron, which tells us nothing about the orbital.

EXAMPLE 7.6

List the values of n, l, and m_l for orbitals in the $2p$ subshell.

Solution: The number given in the designation of the subshell is the principal quantum number, so in this case $n = 2$. For p orbitals, $l = 1$. m_l can have integer values from $-l$ to $+l$. Therefore, m_l can be -1, 0, and $+1$. (The three values for m_l correspond to the three p orbitals.)

PRACTICE EXERCISE

6. List the values of n, l, and m_l for orbitals in the $4f$ subshell.

Text Problems: 7.56, 7.62

B. Counting the number of orbitals associated with a principal quantum number

Strategy: To work this type of problem, you must take into account the energy level (n), the types of orbitals in that energy level (l), and the number of orbitals in a subshell with a particular l value (m_l).

EXAMPLE 7.7

What is the total number of orbitals associated with the principal quantum number $n = 2$?

Solution: For $n = 2$, there are only two possible values of l, 0 and 1. Thus, there is one $2s$ orbital, and there are three $2p$ orbitals. (For $l = 1$, there are three possible m_l values, -1, 0, and $+1$.)

Therefore, the total number of orbitals in the $n = 2$ energy level is $1 + 3 = 4$.

> **Tip:** The total number of orbitals with a given n value is n^2. For Example 7.7, the total number of orbitals in the $n = 2$ level equals $2^2 = 4$.

PRACTICE EXERCISE

7. What is the total number of orbitals associated with the principal quantum number $n = 4$?

Text Problem: 7.62

C. Assigning quantum numbers to an electron

Strategy: In assigning quantum numbers to an electron, you need to specify all four quantum numbers. In most cases, there will be more than one possible set of quantum numbers that can designate an electron.

EXAMPLE 7.8

List the different ways to write the four quantum numbers that designate an electron in a $4s$ orbital.

Solution: To begin with, we know that the principal quantum number n is 4, and the angular momentum quantum number l is 0 (s orbital). For $l = 0$, there is only one possible value for m_l, also 0. Since the electron spin quantum number m_s can be either $+1/2$ or $-1/2$, we conclude that there are two possible ways to designate the electron:

$$(4, 0, 0, +1/2) \qquad (4, 0, 0, -1/2)$$

PRACTICE EXERCISE

8. List the different ways to write the four quantum numbers that designate an electron in a $3d$ orbital.

Text Problems: 7.56, 7.58

D. Counting the number of electrons in a principal level

Strategy: To work this type of problem, you need to know that the number of orbitals with a particular l value is $(2l + 1)$. Also, each orbital can accommodate two electrons.

EXAMPLE 7.9

What is the maximum number of electrons that can be present in the principal level for which $n = 4$?

Solution: When $n = 4$, $l = 0, 1, 2$, and 3. The number of orbitals for each l value is given by

Value of l	Number of orbitals $(2l + 1)$
0	1
1	3
2	5
3	7

The total number of orbitals in the principal level $n = 4$ is sixteen. Since each orbital can accommodate two electrons, the maximum number of electrons that can reside in the orbitals is $2 \times 16 = 32$.

> **Tip:** The above result can be generalized by the formula $2n^2$. For Example 7.9, we have $n = 4$, so $2(4)^2 = 32$.

PRACTICE EXERCISE

9. What is the maximum number of electrons that can be present in the principal level for which $n = 2$?

Text Problem: 7.64

PROBLEM TYPE 6: WRITING ELECTRON CONFIGURATIONS AND ORBITAL DIAGRAMS

The electron configuration of an atom tells us how the electrons are distributed among the various atomic orbitals. To write electron configurations, you should follow the four rules or guidelines given below.

(1) The electron configurations of all elements except hydrogen and helium are represented by a *noble gas core*, which shows (in brackets) the noble gas element that most nearly precedes the element being considered. The noble gas core is followed by the electron configurations of filled or partially filled subshells in the outermost shells.

(2) *The Aufbau or "building up" principle* states that electrons are added to atomic orbitals starting with the lowest energy orbital and "building up" to higher energy orbitals.

(3) In many-electron atoms, the subshells are filled in the order shown in Figure 7.24 of the text.

(4) Each orbital can hold only *two* electrons.

The electron configuration can be represented in a more detailed manner called an ***orbital diagram*** that shows the spin of the electron. For orbital diagrams, you need to follow two additional rules given below.

(5) The *Pauli exclusion principle* states that no two electrons in an atom can have the same four quantum numbers. This means that electrons occupying the same orbital *cannot* have the same spin.

(6) *Hund's rule* states that the most stable arrangement of electrons in subshells is the one with the greatest number of parallel spins, without violating the Pauli exclusion principle.

EXAMPLE 7.10

Write the ground-state electron configuration and the orbital diagram for selenium.

Strategy: How many electrons are in the Se atom (Z = 34)? We start with $n = 1$ and proceed to fill orbitals in the order shown in Figure 7.23 of the text. Remember that any given orbital can hold at most 2 electrons. However, don't forget about degenerate orbitals. Starting with $n = 2$, there are three p orbitals of equal energy, corresponding to $m_l = -1, 0, 1$. Starting with $n = 3$, there are five d orbitals of equal energy, corresponding to $m_l = -2, -1, 0, 1, 2$. We can place electrons in the orbitals according to the Pauli exclusion principle and Hund's rule. The task is simplified if we use the noble gas core preceding Se for the inner electrons.

Solution: Selenium has 34 electrons. The noble gas core in this case is [Ar]. (Ar is the noble gas in the period preceding germanium.) [Ar] represents $1s^22s^22p^63s^23p^6$. This core accounts for 18 electrons, which leaves 16 electrons to place.

See Figure 7.23 of your text to check the order of filling subshells past the Ar noble gas core. You should find that the order of filling is $4s$, $3d$, $4p$. There are 16 remaining electrons to distribute among these orbitals. The $4s$ orbital can hold two electrons. Each of the five $3d$ orbitals can hold two electrons for a total of *10* electrons. This leaves four electrons to place in the $4p$ orbitals.

The electrons configuration for Se is:

$$[\mathrm{Ar}]\ 4s^23d^{10}4p^4$$

To write an *orbital diagram*, we must also specify the spin of the electrons. The $4s$ and $3d$ orbitals are filled, so according to the Pauli exclusion principle, the paired electrons in the $4s$ orbital and in each of the $3d$ orbitals *must* have opposite spins.

$$[\mathrm{Ar}]\ \underset{4s^2}{\underline{\uparrow\downarrow}}\qquad \underset{3d^{10}}{\underline{\uparrow\downarrow}\ \underline{\uparrow\downarrow}\ \underline{\uparrow\downarrow}\ \underline{\uparrow\downarrow}\ \underline{\uparrow\downarrow}}$$

Now, let's deal with the $4p$ electrons. Hund's rule states that the most stable arrangement of electrons in subshells is the one with the greatest number of parallel spins. In other words, we want to keep electrons unpaired if possible with parallel spins. Since there are three p orbitals, three of the p electrons can be placed individually in each of the p subshells with parallel spins.

$$\underset{4p^3}{\underline{\uparrow}\ \underline{\uparrow}\ \underline{\uparrow}}$$

Finally, the fourth p electron must be paired up in one of the $4p$ orbitals. The complete orbital diagram is:

$$[\mathrm{Ar}]\ \underset{4s^2}{\underline{\uparrow\downarrow}}\qquad \underset{3d^{10}}{\underline{\uparrow\downarrow}\ \underline{\uparrow\downarrow}\ \underline{\uparrow\downarrow}\ \underline{\uparrow\downarrow}\ \underline{\uparrow\downarrow}}\qquad \underset{4p^4}{\underline{\uparrow\downarrow}\ \underline{\uparrow}\ \underline{\uparrow}}$$

PRACTICE EXERCISE

10. Write the electron configuration and the orbital diagram for iron (Fe).

Text Problems: 7.76, 7.78, **7.90**, 7.92

ANSWERS TO PRACTICE EXERCISES

1. $\lambda = 0.1224$ m

2. $E = 2.97 \times 10^{-19}$ J
$E = 1.79 \times 10^{5}$ J/mol

3. $\lambda = 1.20 \times 10^{-6}$ m $= 1.20 \times 10^{3}$ nm

4. $n_i = 5$

5. $\lambda = 1.5 \times 10^{-10}$ m

6. $n = 4$, $l = 3$, $m_l = -3, -2, -1, 0, 1, 2, 3$

7. Total number of orbitals $= n^2 = 4^2 = 16$

8.

(3, 2, −2, +1/2)	(3, 2, −2, −1/2)
(3, 2, −1, +1/2)	(3, 2, −1, −1/2)
(3, 2, 0, +1/2)	(3, 2, 0, −1/2)
(3, 2, +1, +1/2)	(3, 2, +1, −1/2)
(3, 2, +2, +1/2)	(3, 2, +2, −1/2)

9. 8 electrons

10. $[Ar]4s^23d^6$

[Ar]	↑↓	↑↓ ↑ ↑ ↑ ↑
	$4s^2$	$3d^6$

SOLUTIONS TO SELECTED TEXT PROBLEMS

7.8 Calculating the Frequency and Wavelength of an Electromagnetic Wave, Problem Type 1.

(a)
Strategy: We are given the wavelength of an electromagnetic wave and asked to calculate the frequency. Rearranging Equation (7.1) of the text and replacing u with c (the speed of light) gives:

$$\nu = \frac{c}{\lambda}$$

Solution: Because the speed of light is given in meters per second, it is convenient to first convert wavelength to units of meters. Recall that 1 nm = 1×10^{-9} m (see Table 1.3 of the text). We write:

$$456\ \cancel{\text{nm}} \times \frac{1 \times 10^{-9}\ \text{m}}{1\ \cancel{\text{nm}}} = 456 \times 10^{-9}\ \text{m} = 4.56 \times 10^{-7}\ \text{m}$$

Substituting in the wavelength and the speed of light (3.00×10^{8} m/s), the frequency is:

$$\nu = \frac{c}{\lambda} = \frac{3.00 \times 10^{8} \frac{\cancel{\text{m}}}{\text{s}}}{4.56 \times 10^{-7}\ \cancel{\text{m}}} = \mathbf{6.58 \times 10^{14}\ s^{-1}}\ \text{or}\ \mathbf{6.58 \times 10^{14}\ Hz}$$

Check: The answer shows that 6.58×10^{14} waves pass a fixed point every second. This very high frequency is in accordance with the very high speed of light.

(b)
Strategy: We are given the frequency of an electromagnetic wave and asked to calculate the wavelength. Rearranging Equation (7.1) of the text and replacing u with c (the speed of light) gives:

$$\lambda = \frac{c}{\nu}$$

Solution: Substituting in the frequency and the speed of light (3.00×10^{8} m/s) into the above equation, the wavelength is:

$$\lambda = \frac{c}{\nu} = \frac{3.00 \times 10^{8} \frac{\text{m}}{\cancel{\text{s}}}}{2.45 \times 10^{9} \frac{1}{\cancel{\text{s}}}} = 0.122\ \text{m}$$

The problem asks for the wavelength in units of nanometers. Recall that 1 nm = 1×10^{-9} m.

$$\lambda = 0.122\ \cancel{\text{m}} \times \frac{1\ \text{nm}}{1 \times 10^{-9}\ \cancel{\text{m}}} = \mathbf{1.22 \times 10^{8}\ nm}$$

7.10 A radio wave is an electromagnetic wave, which travels at the speed of light. The speed of light is in units of m/s, so let's convert distance from units of miles to meters. (28 million mi = 2.8×10^{7} mi)

$$?\ \text{distance (m)} = (2.8 \times 10^{7}\ \cancel{\text{mi}}) \times \frac{1.61\ \cancel{\text{km}}}{1\ \cancel{\text{mi}}} \times \frac{1000\ \text{m}}{1\ \cancel{\text{km}}} = 4.5 \times 10^{10}\ \text{m}$$

Now, we can use the speed of light as a conversion factor to convert from meters to seconds ($c = 3.00 \times 10^8$ m/s).

$$\textbf{? min} = (4.5 \times 10^{10}\ \cancel{\text{m}}) \times \frac{1\ \text{s}}{3.00 \times 10^{8}\ \cancel{\text{m}}} = 1.5 \times 10^{2}\ \text{s} = \mathbf{2.5\ min}$$

7.12 The wavelength is:

$$\lambda = \frac{1\ \text{m}}{1{,}650{,}763.73\ \text{wavelengths}} = 6.05780211 \times 10^{-7}\ \text{m}$$

$$\nu = \frac{c}{\lambda} = \frac{3.00 \times 10^{8}\ \cancel{\text{m}}/\text{s}}{6.05780211 \times 10^{-7}\ \cancel{\text{m}}} = \mathbf{4.95 \times 10^{14}\ s^{-1}}$$

7.16 Calculating the wavelength of electromagnetic radiation and calculating the energy of a photon, Problem Types 1 and 2.

(a)
Strategy: We are given the frequency of an electromagnetic wave and asked to calculate the wavelength. Rearranging Equation (7.1) of the text and replacing u with c (the speed of light) gives:

$$\lambda = \frac{c}{\nu}$$

Solution: Substituting in the frequency and the speed of light (3.00×10^8 m/s) into the above equation, the wavelength is:

$$\lambda = \frac{3.00 \times 10^{8}\ \frac{\text{m}}{\cancel{\text{s}}}}{7.5 \times 10^{14}\ \frac{1}{\cancel{\text{s}}}} = 4.0 \times 10^{-7}\ \text{m} = \mathbf{4.0 \times 10^{2}\ nm}$$

Check: The wavelength of 400 nm calculated is in the blue region of the visible spectrum as expected.

(b)
Strategy: We are given the frequency of an electromagnetic wave and asked to calculate its energy. Equation (7.2) of the text relates the energy and frequency of an electromagnetic wave.

$$E = h\nu$$

Solution: Substituting in the frequency and Planck's constant (6.63×10^{-34} J·s) into the above equation, the energy of a single photon associated with this frequency is:

$$E = h\nu = (6.63 \times 10^{-34}\ \text{J}\cdot\cancel{\text{s}})\left(7.5 \times 10^{14}\ \frac{1}{\cancel{\text{s}}}\right) = \mathbf{5.0 \times 10^{-19}\ J}$$

Check: We expect the energy of a single photon to be a very small energy as calculated above, 5.0×10^{-19} J.

7.18 The energy given in this problem is for *1 mole* of photons. To apply $E = h\nu$, we must divide the energy by Avogadro's number. The energy of one photon is:

$$E = \frac{1.0 \times 10^{3}\ \cancel{\text{kJ}}}{1\ \cancel{\text{mol}}} \times \frac{1\ \cancel{\text{mol}}}{6.022 \times 10^{23}\ \text{photons}} \times \frac{1000\ \text{J}}{1\ \cancel{\text{kJ}}} = 1.7 \times 10^{-18}\ \text{J/photon}$$

The wavelength of this photon can be found using the relationship, $E = \dfrac{hc}{\lambda}$.

$$\lambda = \frac{hc}{E} = \frac{(6.63\times10^{-34}\,\cancel{\text{J}}\cdot\cancel{\text{s}})\left(3.00\times10^{8}\,\frac{\text{m}}{\cancel{\text{s}}}\right)}{1.7\times10^{-18}\,\cancel{\text{J}}} = 1.2\times10^{-7}\,\cancel{\text{m}}\times\frac{1\text{ nm}}{1\times10^{-9}\,\cancel{\text{m}}} = \mathbf{1.2\times10^{2}\ nm}$$

The radiation is in the **ultraviolet** region (see Figure 7.4 of the text).

7.20 (a) $\lambda = \dfrac{c}{\nu}$

$$\lambda = \frac{3.00\times10^{8}\,\frac{\text{m}}{\cancel{\text{s}}}}{8.11\times10^{14}\,\frac{1}{\cancel{\text{s}}}} = 3.70\times10^{-7}\text{ m} = \mathbf{3.70\times10^{2}\ nm}$$

(b) Checking Figure 7.4 of the text, you should find that the visible region of the spectrum runs from 400 to 700 nm. 370 nm is in the **ultraviolet** region of the spectrum.

(c) $E = h\nu$. Substitute the frequency (ν) into this equation to solve for the energy of one quantum associated with this frequency.

$$E = h\nu = (6.63\times10^{-34}\,\text{J}\cdot\cancel{\text{s}})\left(8.11\times10^{14}\,\frac{1}{\cancel{\text{s}}}\right) = \mathbf{5.38\times10^{-19}\ J}$$

7.26 The emitted light could be analyzed by passing it through a prism.

7.28 Excited atoms of the chemical elements emit the same characteristic frequencies or lines in a terrestrial laboratory, in the sun, or in a star many light-years distant from earth.

7.30 We use more accurate values of h and c for this problem.

$$E = \frac{hc}{\lambda} = \frac{(6.6256\times10^{-34}\,\text{J}\cdot\cancel{\text{s}})(2.998\times10^{8}\,\cancel{\text{m}}/\cancel{\text{s}})}{656.3\times10^{-9}\,\cancel{\text{m}}} = \mathbf{3.027\times10^{-19}\ J}$$

7.32 Calculating the Energy, Wavelength, or Frequency in the Emission Spectrum of a Hydrogen Atom, Problem Type 3.

Strategy: We are given the initial and final states in the emission process. We can calculate the energy of the emitted photon using Equation (7.6) of the text. Then, from this energy, we can solve for the frequency of the photon, and from the frequency we can solve for the wavelength. The value of Rydberg's constant is 2.18×10^{-18} J.

Solution: From Equation (7.6) we write:

$$\Delta E = R_{\text{H}}\left(\frac{1}{n_{\text{i}}^{2}} - \frac{1}{n_{\text{f}}^{2}}\right)$$

$$\Delta E = (2.18\times10^{-18}\,\text{J})\left(\frac{1}{4^{2}} - \frac{1}{2^{2}}\right)$$

$$\Delta E = -4.09\times10^{-19}\,\text{J}$$

The negative sign for ΔE indicates that this is energy associated with an emission process. To calculate the frequency, we will omit the minus sign for ΔE because the frequency of the photon must be positive. We know that

$$\Delta E = h\nu$$

Rearranging the equation and substituting in the known values,

$$\nu = \frac{\Delta E}{h} = \frac{(4.09 \times 10^{-19}\ \cancel{\text{J}})}{(6.63 \times 10^{-34}\ \cancel{\text{J}}\cdot\text{s})} = \mathbf{6.17 \times 10^{14}\ s^{-1}} \text{ or } \mathbf{6.17 \times 10^{14}\ Hz}$$

We also know that $\lambda = \frac{c}{\nu}$. Substituting the frequency calculated above into this equation gives:

$$\lambda = \frac{3.00 \times 10^{8}\ \frac{\text{m}}{\cancel{\text{s}}}}{\left(6.17 \times 10^{14}\ \frac{1}{\cancel{\text{s}}}\right)} = 4.86 \times 10^{-7}\ \text{m} = \mathbf{486\ nm}$$

Check: This wavelength is in the visible region of the electromagnetic region (see Figure 7.4 of the text). This is consistent with the fact that because $n_i = 4$ and $n_f = 2$, this transition gives rise to a spectral line in the Balmer series (see Figure 7.6 of the text).

7.34 $$\Delta E = R_H\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)$$

n_f is given in the problem and R_H is a constant, but we need to calculate ΔE. The photon energy is:

$$E = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34}\ \text{J}\cdot\cancel{\text{s}})(3.00 \times 10^{8}\ \cancel{\text{m}}/\cancel{\text{s}})}{434 \times 10^{-9}\ \cancel{\text{m}}} = 4.58 \times 10^{-19}\ \text{J}$$

Since this is an emission process, the energy change ΔE must be negative, or -4.58×10^{-19} J.

Substitute ΔE into the following equation, and solve for n_i.

$$\Delta E = R_H\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)$$

$$-4.58 \times 10^{-19}\ \text{J} = (2.18 \times 10^{-18}\ \text{J})\left(\frac{1}{n_i^2} - \frac{1}{2^2}\right)$$

$$\frac{1}{n_i^2} = \left(\frac{-4.58 \times 10^{-19}\ \cancel{\text{J}}}{2.18 \times 10^{-18}\ \cancel{\text{J}}}\right) + \frac{1}{2^2} = -0.210 + 0.250 = 0.040$$

$$n_i = \frac{1}{\sqrt{0.040}} = \mathbf{5}$$

7.40 The de Broglie Equation: Calculating the Wavelengths of Particles, Problem Type 4.

Strategy: We are given the mass and the speed of the proton and asked to calculate the wavelength. We need the de Broglie equation, which is Equation (7.8) of the text. Note that because the units of Planck's constant are J·s, m must be in kg and u must be in m/s (1 J = 1 kg·m²/s²).

Solution: Using Equation (7.8) we write:

$$\lambda = \frac{h}{mu}$$

$$\lambda = \frac{h}{mu} = \frac{\left(6.63 \times 10^{-34} \frac{\text{kg}\cdot\text{m}^2}{\text{s}^2}\cdot\text{s}\right)}{(1.673 \times 10^{-27}\ \text{kg})(2.90 \times 10^8\ \text{m/s})} = 1.37 \times 10^{-15}\ \text{m}$$

The problem asks to express the wavelength in nanometers.

$$\lambda = (1.37 \times 10^{-15}\ \text{m}) \times \frac{1\ \text{nm}}{1 \times 10^{-9}\ \text{m}} = \mathbf{1.37 \times 10^{-6}\ nm}$$

7.42 First, we convert mph to m/s.

$$\frac{35\ \text{mi}}{1\ \text{h}} \times \frac{1.61\ \text{km}}{1\ \text{mi}} \times \frac{1000\ \text{m}}{1\ \text{km}} \times \frac{1\ \text{h}}{3600\ \text{s}} = 16\ \text{m/s}$$

$$\lambda = \frac{h}{mu} = \frac{\left(6.63 \times 10^{-34} \frac{\text{kg}\cdot\text{m}^2}{\text{s}^2}\cdot\text{s}\right)}{(2.5 \times 10^{-3}\ \text{kg})(16\ \text{m/s})} = 1.7 \times 10^{-32}\ \text{m} = \mathbf{1.7 \times 10^{-23}\ nm}$$

7.56 Quantum Numbers, Problem Type 5.

Strategy: What are the relationships among n, l, and m_l?

Solution: We are given the principal quantum number, $n = 3$. The possible l values range from 0 to $(n - 1)$. Thus, there are three possible values of l: 0, 1, and 2, corresponding to the s, p, and d orbitals, respectively. The values of m_l can vary from $-l$ to l. The values of m_l for each l value are:

$l = 0$: $m_l = 0$ $l = 1$: $m_l = -1, 0, 1$ $l = 2$: $m_l = -2, -1, 0, 1, 2$

7.58 **(a)** The number given in the designation of the subshell is the principal quantum number, so in this case $\boldsymbol{n = 3}$. For s orbitals, $\boldsymbol{l = 0}$. m_l can have integer values from $-l$ to $+l$, therefore, $\boldsymbol{m_l = 0}$. The electron spin quantum number, $\boldsymbol{m_s}$, can be either **+1/2** or **−1/2**.

Following the same reasoning as part **(a)**

(b) 4p: $n = 4$; $l = 1$; $m_l = -1, 0, 1$; $m_s = +1/2, -1/2$

(c) 3d: $n = 3$; $l = 2$; $m_l = -2, -1, 0, 1, 2$; $m_s = +1/2, -1/2$

7.60 The two orbitals are identical in size, shape, and energy. They differ only in their orientation with respect to each other.

Can you assign a specific value of the magnetic quantum number to these orbitals? What are the allowed values of the magnetic quantum number for the $2p$ subshell?

7.62 For $n = 6$, the allowed values of l are 0, 1, 2, 3, 4, and 5 [$l = 0$ to $(n - 1)$, integer values]. These l values correspond to the $6s$, $6p$, $6d$, $6f$, $6g$, and $6h$ subshells. These subshells each have 1, 3, 5, 7, 9, and 11 orbitals, respectively (number of orbitals = $2l + 1$).

7.64

n value	orbital sum	total number of electrons
1	1	2
2	$1 + 3 = 4$	8
3	$1 + 3 + 5 = 9$	18
4	$1 + 3 + 5 + 7 = 16$	32
5	$1 + 3 + 5 + 7 + 9 = 25$	50
6	$1 + 3 + 5 + 7 + 9 + 11 = 36$	72

In each case the total number of orbitals is just the square of the n value (n^2). The total number of electrons is $\mathbf{2n^2}$.

7.66 The electron configurations for the elements are

(a) N: $1s^22s^22p^3$ There are three p-type electrons.

(b) Si: $1s^22s^22p^63s^23p^2$ There are six s-type electrons.

(c) S: $1s^22s^22p^63s^23p^4$ There are no d-type electrons.

7.68 In the many-electron atom, the $3p$ orbital electrons are more effectively shielded by the inner electrons of the atom (that is, the $1s$, $2s$, and $2p$ electrons) than the $3s$ electrons. The $3s$ orbital is said to be more "penetrating" than the $3p$ and $3d$ orbitals. In the hydrogen atom there is only one electron, so the $3s$, $3p$, and $3d$ orbitals have the same energy.

7.70 **(a)** $2s < 2p$ **(b)** $3p < 3d$ **(c)** $3s < 4s$ **(d)** $4d < 5f$

7.76 For aluminum, there are not enough electrons in the $2p$ subshell. (The $2p$ subshell holds six electrons.) The number of electrons (13) is correct. The electron configuration should be $1s^22s^22p^63s^23p^1$. The configuration shown might be an excited state of an aluminum atom.

For boron, there are too many electrons. (Boron only has five electrons.) The electron configuration should be $1s^22s^22p^1$. What would be the electric charge of a boron ion with the electron arrangement given in the problem?

For fluorine, there are also too many electrons. (Fluorine only has nine electrons.) The configuration shown is that of the F^- ion. The correct electron configuration is $1s^22s^22p^5$.

7.78 You should write the electron configurations for each of these elements to answer this question. In some cases, an orbital diagram may be helpful.

B: $[He]2s^22p^1$ (1 unpaired electron)
Ne: (0 unpaired electrons, Why?)
P: $[Ne]3s^23p^3$ (3 unpaired electrons)
Sc: $[Ar]4s^23d^1$ (1 unpaired electron)

Mn: [Ar]$4s^2 3d^5$ (5 unpaired electrons)
Kr: (0 unpaired electrons)
Cd: [Kr]$5s^2 4d^{10}$ (0 unpaired electrons)
Pb: [Xe]$6s^2 4f^{14} 5d^{10} 6p^2$ (2 unpaired electrons)
Se: [Ar]$4s^2 3d^{10} 4p^4$ (2 unpaired electrons)
Fe: [Ar]$4s^2 3d^6$ (4 unpaired electrons)
I: [Kr]$5s^2 4d^{10} 5p^5$ (1 unpaired electron)

7.88 The ground state electron configuration of Tc is: [Kr]$5s^2 4d^5$.

7.90 Writing Electron Configurations, Problem Type 6.

Strategy: How many electrons are in the Ge atom (Z = 32)? We start with $n = 1$ and proceed to fill orbitals in the order shown in Figure 7.23 of the text. Remember that any given orbital can hold at most 2 electrons. However, don't forget about degenerate orbitals. Starting with $n = 2$, there are three p orbitals of equal energy, corresponding to $m_l = -1, 0, 1$. Starting with $n = 3$, there are five d orbitals of equal energy, corresponding to $m_l = -2, -1, 0, 1, 2$. We can place electrons in the orbitals according to the Pauli exclusion principle and Hund's rule. The task is simplified if we use the noble gas core preceding Ge for the inner electrons.

Solution: Germanium has 32 electrons. The noble gas core in this case is [Ar]. (Ar is the noble gas in the period preceding germanium.) [Ar] represents $1s^2 2s^2 2p^6 3s^2 3p^6$. This core accounts for 18 electrons, which leaves 14 electrons to place.

See Figure 7.23 of your text to check the order of filling subshells past the Ar noble gas core. You should find that the order of filling is $4s$, $3d$, $4p$. There are 14 remaining electrons to distribute among these orbitals. The $4s$ orbital can hold two electrons. Each of the five $3d$ orbitals can hold two electrons for a total of *10* electrons. This leaves two electrons to place in the $4p$ orbitals.

The electrons configuration for Ge is:

$$[Ar]4s^2 3d^{10} 4p^2$$

You should follow the same reasoning for the remaining atoms.

Fe: [Ar]$4s^2 3d^6$
Zn: [Ar]$4s^2 3d^{10}$
Ni: [Ar]$4s^2 3d^8$
W: [Xe]$6s^2 4f^{14} 5d^4$
Tl: [Xe]$6s^2 4f^{14} 5d^{10} 6p^1$

7.92

S^+	S	S^-
$\uparrow\downarrow$ $3s^2$ $\uparrow$ $\uparrow$ $\uparrow$ $3p^3$	$\uparrow\downarrow$ $3s^2$ $\uparrow\downarrow$ $\uparrow$ $\uparrow$ $3p^4$	$\uparrow\downarrow$ $3s^2$ $\uparrow\downarrow$ $\uparrow\downarrow$ $\uparrow$ $3p^5$
S^+ (5 valence electrons) 3 unpaired electrons	S (6 valence electrons) 2 unpaired electrons	S^- (7 valence electrons) 1 unpaired electron

S^+ has the most unpaired electrons

7.94 Part **(b)** is correct in the view of contemporary quantum theory. Bohr's explanation of emission and absorption line spectra appears to have universal validity. Parts **(a)** and **(c)** are artifacts of Bohr's early planetary model of the hydrogen atom and are *not* considered to be valid today.

7.96 (a) With $n = 2$, there are n^2 orbitals $= 2^2 = 4$. $m_s = +1/2$, specifies 1 electron per orbital, for a total of **4 electrons**.

(b) $n = 4$ and $m_l = +1$, specifies one orbital in each subshell with $l = 1, 2$, or 3 (i.e., a $4p$, $4d$, and $4f$ orbital). Each of the three orbitals holds 2 electrons for a total of **6 electrons.**

(c) If $n = 3$ and $l = 2$, m_l has the values 2, 1, 0, −1, or −2. Each of the five orbitals can hold 2 electrons for a total of **10 electrons** (2 e^- in each of the five $3d$ orbitals).

(d) If $n = 2$ and $l = 0$, then m_l can only be zero. $m_s = -1/2$ specifies 1 electron in this orbital for a total of **1 electron** (one e^- in the $2s$ orbital).

(e) $n = 4$, $l = 3$ and $m_l = -2$, specifies one $4f$ orbital. This orbital can hold **2 electrons.**

7.98 The wave properties of electrons are used in the operation of an electron microscope.

7.100 **(a)** First convert 100 mph to units of m/s.

$$\frac{100\text{ mi}}{1\text{ h}} \times \frac{1\text{ h}}{3600\text{ s}} \times \frac{1.609\text{ km}}{1\text{ mi}} \times \frac{1000\text{ m}}{1\text{ km}} = 44.7\text{ m/s}$$

Using the de Broglie equation:

$$\lambda = \frac{h}{mu} = \frac{\left(6.63 \times 10^{-34}\ \frac{\text{kg}\cdot\text{m}^2}{\text{s}^2}\cdot\text{s}\right)}{(0.141\text{ kg})(44.7\text{ m/s})} = 1.05 \times 10^{-34}\text{ m} = \mathbf{1.05 \times 10^{-25}\ nm}$$

(b) The average mass of a hydrogen atom is:

$$\frac{1.008\text{ g}}{1\text{ mol}} \times \frac{1\text{ mol}}{6.022 \times 10^{23}\text{ atoms}} = 1.674 \times 10^{-24}\text{ g/H atom} = 1.674 \times 10^{-27}\text{ kg}$$

$$\lambda = \frac{h}{mu} = \frac{\left(6.63 \times 10^{-34}\ \frac{\text{kg}\cdot\text{m}^2}{\text{s}^2}\cdot\text{s}\right)}{(1.674 \times 10^{-27}\text{ kg})(44.7\text{ m/s})} = 8.86 \times 10^{-9}\text{ m} = \mathbf{8.86\ nm}$$

7.102 **(a)** First, we can calculate the energy of a single photon with a wavelength of 633 nm.

$$E = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34}\text{ J}\cdot\text{s})(3.00 \times 10^8\text{ m/s})}{633 \times 10^{-9}\text{ m}} = 3.14 \times 10^{-19}\text{ J}$$

The number of photons produced in a 0.376 J pulse is:

$$0.376\text{ J} \times \frac{1\text{ photon}}{3.14 \times 10^{-19}\text{ J}} = \mathbf{1.20 \times 10^{18}\ photons}$$

(b) Since a 1 W = 1 J/s, the power delivered per a 1.00×10^{-9} s pulse is:

$$\frac{0.376\text{ J}}{1.00 \times 10^{-9}\text{ s}} = 3.76 \times 10^8\text{ J/s} = \mathbf{3.76 \times 10^8\ W}$$

Compare this with the power delivered by a 100-W light bulb!

7.104 First, let's find the energy needed to photodissociate one water molecule.

$$\frac{285.8 \text{ kJ}}{1 \text{ mol}} \times \frac{1 \text{ mol}}{6.022 \times 10^{23} \text{ molecules}} = 4.746 \times 10^{-22} \text{ kJ/molecule} = 4.746 \times 10^{-19} \text{ J/molecule}$$

The maximum wavelength of a photon that would provide the above energy is:

$$\lambda = \frac{hc}{E} = \frac{(6.63 \times 10^{-34} \text{ J}\cdot\text{s})(3.00 \times 10^{8} \text{ m/s})}{4.746 \times 10^{-19} \text{ J}} = \mathbf{4.19 \times 10^{-7}\ m} = \mathbf{419\ nm}$$

This wavelength is in the visible region of the electromagnetic spectrum. Since water is continuously being struck by visible radiation *without* decomposition, it seems unlikely that photodissociation of water by this method is feasible.

7.106 Since 1 W = 1 J/s, the energy output of the light bulb in 1 second is 75 J. The actual energy converted to visible light is 15 percent of this value or 11 J.

First, we need to calculate the energy of one 550 nm photon. Then, we can determine how many photons are needed to provide 11 J of energy.

The energy of one 550 nm photon is:

$$E = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34} \text{ J}\cdot\text{s})(3.00 \times 10^{8} \text{ m/s})}{550 \times 10^{-9} \text{ m}} = 3.62 \times 10^{-19} \text{ J/photon}$$

The number of photons needed to produce 11 J of energy is:

$$11 \text{ J} \times \frac{1 \text{ photon}}{3.62 \times 10^{-19} \text{ J}} = \mathbf{3.0 \times 10^{19}\ photons}$$

7.108 The Balmer series corresponds to transitions to the $n = 2$ level.

For He^{+}:

$$\Delta E = R_{\text{He}^+}\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right) \qquad \lambda = \frac{hc}{\Delta E} = \frac{(6.63 \times 10^{-34} \text{ J}\cdot\text{s})(3.00 \times 10^{8} \text{ m/s})}{\Delta E}$$

For the transition, $n = 3 \rightarrow 2$

$$\Delta E = (8.72 \times 10^{-18} \text{ J})\left(\frac{1}{3^2} - \frac{1}{2^2}\right) = -1.21 \times 10^{-18} \text{ J} \qquad \lambda = \frac{1.99 \times 10^{-25} \text{ J}\cdot\text{m}}{1.21 \times 10^{-18} \text{ J}} = 1.64 \times 10^{-7} \text{ m} = \mathbf{164\ nm}$$

For the transition, $n = 4 \rightarrow 2$, $\Delta E = -1.64 \times 10^{-18}$ J **λ = 121 nm**

For the transition, $n = 5 \rightarrow 2$, $\Delta E = -1.83 \times 10^{-18}$ J **λ = 109 nm**

For the transition, $n = 6 \rightarrow 2$, $\Delta E = -1.94 \times 10^{-18}$ J **λ = 103 nm**

For H, the calculations are identical to those above, except the Rydberg constant for H is 2.18×10^{-18} J.

For the transition, $n = 3 \rightarrow 2$, $\Delta E = -3.03 \times 10^{-19}$ J **λ = 657 nm**

For the transition, $n = 4 \rightarrow 2$, $\Delta E = -4.09 \times 10^{-19}$ J **λ = 487 nm**

For the transition, $n = 5 \rightarrow 2$, $\Delta E = -4.58 \times 10^{-19}$ J $\lambda =$ **434 nm**

For the transition, $n = 6 \rightarrow 2$, $\Delta E = -4.84 \times 10^{-19}$ J $\lambda =$ **411 nm**

All the Balmer transitions for He^+ are in the ultraviolet region; whereas, the transitions for H are all in the visible region. Note the negative sign for energy indicating that a photon has been emitted.

7.110 First, we need to calculate the energy of one 600 nm photon. Then, we can determine how many photons are needed to provide 4.0×10^{-17} J of energy.

The energy of one 600 nm photon is:

$$E = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34}\ \text{J}\cdot\text{s})(3.00 \times 10^{8}\ \text{m/s})}{600 \times 10^{-9}\ \text{m}} = 3.32 \times 10^{-19}\ \text{J/photon}$$

The number of photons needed to produce 4.0×10^{-17} J of energy is:

$$(4.0 \times 10^{-17}\ \text{J}) \times \frac{1\ \text{photon}}{3.32 \times 10^{-19}\ \text{J}} = \mathbf{1.2 \times 10^{2}\ photons}$$

7.112 A "blue" photon (shorter wavelength) is higher energy than a "yellow" photon. For the same amount of energy delivered to the metal surface, there must be fewer "blue" photons than "yellow" photons. Thus, the yellow light would eject more electrons since there are more "yellow" photons. Since the "blue" photons are of higher energy, blue light will eject electrons with greater kinetic energy.

7.114 The excited atoms are still neutral, so the total number of electrons is the same as the atomic number of the element.

(a) He (2 electrons), $1s^2$
(b) N (7 electrons), $1s^22s^22p^3$
(c) Na (11 electrons), $1s^22s^22p^63s^1$
(d) As (33 electrons), $[Ar]4s^23d^{10}4p^3$
(e) Cl (17 electrons), $[Ne]3s^23p^5$

7.116 Rutherford and his coworkers might have discovered the wave properties of electrons.

7.118 The wavelength of a He atom can be calculated using the de Broglie equation. First, we need to calculate the root-mean-square speed using Equation (5.16) from the text.

$$u_{rms} = \sqrt{\frac{3\left(8.314\frac{\text{J}}{\text{K}\cdot\text{mol}}\right)(273 + 20)\text{K}}{4.003 \times 10^{-3}\ \text{kg/mol}}} = 1.35 \times 10^{3}\ \text{m/s}$$

To calculate the wavelength, we also need the mass of a He atom in kg.

$$\frac{4.003 \times 10^{-3}\ \text{kg He}}{1\ \text{mol He}} \times \frac{1\ \text{mol He}}{6.022 \times 10^{23}\ \text{He atoms}} = 6.647 \times 10^{-27}\ \text{kg/atom}$$

Finally, the wavelength of a He atom is:

$$\lambda = \frac{h}{mu} = \frac{(6.63 \times 10^{-34}\ \text{J}\cdot\text{s})}{(6.647 \times 10^{-27}\ \text{kg})(1.35 \times 10^{3}\ \text{m/s})} = \mathbf{7.39 \times 10^{-11}\ m} = \mathbf{7.39 \times 10^{-2}\ nm}$$

7.120 (a) **False.** $n = 2$ is the first excited state.

(b) **False.** In the $n = 4$ state, the electron is (on average) further from the nucleus and hence easier to remove.

(c) **True.**

(d) **False.** The $n = 4$ to $n = 1$ transition is a higher energy transition, which corresponds to a *shorter* wavelength.

(e) **True.**

7.122 We use Heisenberg's uncertainty principle with the equality sign to calculate the minimum uncertainty.

$$\Delta x \Delta p = \frac{h}{4\pi}$$

The momentum (p) is equal to the mass times the velocity.

$$p = mu \qquad \text{or} \qquad \Delta p = m\Delta u$$

We can write:

$$\Delta p = m\Delta u = \frac{h}{4\pi \Delta x}$$

Finally, the uncertainty in the velocity of the oxygen molecule is:

$$\Delta u = \frac{h}{4\pi m \Delta x} = \frac{(6.63 \times 10^{-34}\ \text{J}\cdot\text{s})}{4\pi(5.3 \times 10^{-26}\ \text{kg})(5.0 \times 10^{-5}\ \text{m})} = \mathbf{2.0 \times 10^{-5}\ m/s}$$

7.124 The Pauli exclusion principle states that no two electrons in an atom can have the same four quantum numbers. In other words, only two electrons may exist in the same atomic orbital, and these electrons must have opposite spins. **(a)** and **(f)** violate the Pauli exclusion principle.

Hund's rule states that the most stable arrangement of electrons in subshells is the one with the greatest number of parallel spins. **(b)**, **(d)**, and **(e)** violate Hund's rule.

7.126 As an estimate, we can equate the energy for ionization ($Fe^{13+} \rightarrow Fe^{14+}$) to the average kinetic energy $\left(\frac{3}{2}RT\right)$ of the ions.

$$\frac{3.5 \times 10^4\ \cancel{\text{kJ}}}{1\ \text{mol}} \times \frac{1000\ \text{J}}{1\ \cancel{\text{kJ}}} = 3.5 \times 10^7\ \text{J}$$

$$IE = \frac{3}{2}RT$$

$$3.5 \times 10^7\ \cancel{\text{J}}/\cancel{\text{mol}} = \frac{3}{2}(8.314\ \cancel{\text{J}}/\cancel{\text{mol}}\cdot\text{K})T$$

$$\mathbf{T = 2.8 \times 10^6\ K}$$

The actual temperature can be, and most probably is, higher than this.

7.128 Looking at the de Broglie equation $\lambda = \frac{h}{mu}$, the mass of an N_2 molecule (in kg) and the velocity of an N_2 molecule (in m/s) is needed to calculate the de Broglie wavelength of N_2.

First, calculate the root-mean-square velocity of N_2.

$\mathcal{M}(N_2) = 28.02\text{ g/mol} = 0.02802\text{ kg/mol}$

$$u_{\text{rms}}(N_2) = \sqrt{\frac{(3)\left(8.314\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)(300\text{ K})}{\left(0.02802\frac{\text{kg}}{\text{mol}}\right)}} = 516.8\text{ m/s}$$

Second, calculate the mass of one N_2 molecule in kilograms.

$$\frac{28.02\text{ g }N_2}{1\text{ mol }N_2} \times \frac{1\text{ mol }N_2}{6.022\times10^{23}\ N_2\text{ molecules}} \times \frac{1\text{ kg}}{1000\text{ g}} = 4.653\times10^{-26}\text{ kg/molecule}$$

Now, substitute the mass of an N_2 molecule and the root-mean-square velocity into the de Broglie equation to solve for the de Broglie wavelength of an N_2 molecule.

$$\lambda = \frac{h}{mu} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})}{(4.653\times10^{-26}\text{ kg})(516.8\text{ m/s})} = \mathbf{2.76\times10^{-11}\text{ m}}$$

7.130 The kinetic energy acquired by the electrons is equal to the voltage times the charge on the electron. After calculating the kinetic energy, we can calculate the velocity of the electrons (KE = $1/2mu^2$). Finally, we can calculate the wavelength associated with the electrons using the de Broglie equation.

$$\text{KE} = (5.00\times10^3\text{ V})\times\frac{1.602\times10^{-19}\text{ J}}{1\text{ V}} = 8.01\times10^{-16}\text{ J}$$

We can now calculate the velocity of the electrons.

$$\text{KE} = \frac{1}{2}mu^2$$

$$8.01\times10^{-16}\text{ J} = \frac{1}{2}(9.1094\times10^{-31}\text{ kg})u^2$$

$$u = 4.19\times10^7\text{ m/s}$$

Finally, we can calculate the wavelength associated with the electrons using the de Broglie equation.

$$\lambda = \frac{h}{mu}$$

$$\lambda = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})}{(9.1094\times10^{-31}\text{ kg})(4.19\times10^7\text{ m/s})} = \mathbf{1.74\times10^{-11}\text{ m} = 17.4\text{ pm}}$$

7.132 The energy given in the problem is the energy of 1 mole of gamma rays. We need to convert this to the energy of one gamma ray, then we can calculate the wavelength and frequency of this gamma ray.

$$\frac{1.29 \times 10^{11}\ \text{J}}{1\ \text{mol}} \times \frac{1\ \text{mol}}{6.022 \times 10^{23}\ \text{gamma rays}} = 2.14 \times 10^{-13}\ \text{J/gamma ray}$$

Now, we can calculate the wavelength and frequency from this energy.

$$E = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{E} = \frac{(6.63 \times 10^{-34}\ \text{J}\cdot\text{s})(3.00 \times 10^{8}\ \text{m/s})}{2.14 \times 10^{-13}\ \text{J}} = \mathbf{9.29 \times 10^{-13}\ m} = \mathbf{0.929\ pm}$$

and

$$E = h\nu$$

$$\nu = \frac{E}{h} = \frac{2.14 \times 10^{-13}\ \text{J}}{6.63 \times 10^{-34}\ \text{J}\cdot\text{s}} = \mathbf{3.23 \times 10^{20}\ s^{-1}}$$

7.134 **(a)** Line A corresponds to the longest wavelength or lowest energy transition, which is the 3 → 2 transition. Therefore, line B corresponds to the 4 → 2 transition, and line C corresponds to the 5 → 2 transition.

(b) We can derive an equation for the energy change (ΔE) for an electronic transition.

$$E_f = -R_H Z^2\left(\frac{1}{n_f^2}\right) \quad \text{and} \quad E_i = -R_H Z^2\left(\frac{1}{n_i^2}\right)$$

$$\Delta E = E_f - E_i = -R_H Z^2\left(\frac{1}{n_f^2}\right) - \left(-R_H Z^2\left(\frac{1}{n_i^2}\right)\right)$$

$$\Delta E = R_H Z^2\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)$$

Line C corresponds to the 5 → 2 transition. The energy change associated with this transition can be calculated from the wavelength (27.1 nm).

$$E = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34}\ \text{J}\cdot\text{s})(3.00 \times 10^{8}\ \text{m/s})}{(27.1 \times 10^{-9}\ \text{m})} = 7.34 \times 10^{-18}\ \text{J}$$

For the 5 → 2 transition, we now know ΔE, n_i, n_f, and R_H ($R_H = 2.18 \times 10^{-18}$ J). Since this transition corresponds to an emission process, energy is released and ΔE is negative. ($\Delta E = -7.34 \times 10^{-18}$ J). We can now substitute these values into the equation above to solve for Z.

$$\Delta E = R_H Z^2\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)$$

$$-7.34 \times 10^{-18}\ \text{J} = (2.18 \times 10^{-18}\ \text{J})Z^2\left(\frac{1}{5^2} - \frac{1}{2^2}\right)$$

$$-7.34 \times 10^{-18}\ \text{J} = (-4.58 \times 10^{-19})Z^2$$

$$Z^2 = 16.0$$

$$Z = 4$$

Z must be an integer because it represents the atomic number of the parent atom.

Now, knowing the value of Z, we can substitute in n_i and n_f for the 3 → 2 (Line A) and the 4 → 2 (Line B) transitions to solve for ΔE. We can then calculate the wavelength from the energy.

For Line A (3 → 2)

$$\Delta E = R_H Z^2\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right) = (2.18 \times 10^{-18}\ \text{J})(4)^2\left(\frac{1}{3^2} - \frac{1}{2^2}\right)$$

$$\Delta E = -4.84 \times 10^{-18}\ \text{J}$$

$$\lambda = \frac{hc}{E} = \frac{(6.63 \times 10^{-34}\ \text{J}\cdot\text{s})(3.00 \times 10^8\ \text{m/s})}{(4.84 \times 10^{-18}\ \text{J})} = \mathbf{4.11 \times 10^{-8}\ m = 41.1\ nm}$$

For Line B (4 → 2)

$$\Delta E = R_H Z^2\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right) = (2.18 \times 10^{-18}\ \text{J})(4)^2\left(\frac{1}{4^2} - \frac{1}{2^2}\right)$$

$$\Delta E = -6.54 \times 10^{-18}\ \text{J}$$

$$\lambda = \frac{hc}{E} = \frac{(6.63 \times 10^{-34}\ \text{J}\cdot\text{s})(3.00 \times 10^8\ \text{m/s})}{(6.54 \times 10^{-18}\ \text{J})} = \mathbf{3.04 \times 10^{-8}\ m = 30.4\ nm}$$

(c) The value of the final energy state is $n_f = \infty$. Use the equation derived in part (b) to solve for ΔE.

$$\Delta E = R_H Z^2\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right) = (2.18 \times 10^{-18}\ \text{J})(4)^2\left(\frac{1}{4^2} - \frac{1}{\infty^2}\right)$$

$$\Delta E = \mathbf{2.18 \times 10^{-18}\ J}$$

(d) As we move to higher energy levels in an atom or ion, the energy levels get closer together. See Figure 7.11 of the text, which represents the energy levels for the hydrogen atom. Transitions from higher energy levels to the $n = 2$ level will be very close in energy and hence will have similar wavelengths. The lines are so close together that they overlap, forming a continuum. The continuum shows that the electron has been removed from the ion, and we no longer have quantized energy levels associated with the electron. In other words, the energy of the electron can now vary continuously.

7.136 To calculate the energy to remove at electron from the $n = 1$ state and the $n = 5$ state in the Li^{2+} ion, we use the equation derived in Problem 7.134 (b).

$$\Delta E = R_H Z^2\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)$$

For $n_i = 1$, $n_f = \infty$, and Z = 3, we have:

$$\Delta E = (2.18 \times 10^{-18}\ \text{J})(3)^2\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right) = \mathbf{1.96 \times 10^{-17}\ J}$$

For $n_i = 5$, $n_f = \infty$, and Z = 3, we have:

$$\Delta E = (2.18 \times 10^{-18}\text{ J})(3)^2\left(\frac{1}{5^2} - \frac{1}{\infty^2}\right) = \mathbf{7.85 \times 10^{-19}\ J}$$

To calculate the wavelength of the emitted photon in the electronic transition from $n = 5$ to $n = 1$, we first calculate ΔE and then calculate the wavelength.

$$\Delta E = R_H Z^2\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right) = (2.18 \times 10^{-18}\text{ J})(3)^2\left(\frac{1}{5^2} - \frac{1}{1^2}\right) = -1.88 \times 10^{-17}\text{ J}$$

We ignore the minus sign for ΔE in calculating λ.

$$\lambda = \frac{hc}{\Delta E} = \frac{(6.63 \times 10^{-34}\ \cancel{\text{J}\cdot\text{s}})(3.00 \times 10^8\text{ m}/\cancel{\text{s}})}{1.88 \times 10^{-17}\ \cancel{\text{J}}}$$

$$\lambda = \mathbf{1.06 \times 10^{-8}\ m = 10.6\ nm}$$

7.138 We calculate W (the energy needed to remove an electron from the metal) at a wavelength of 351 nm. Once W is known, we can then calculate the velocity of an ejected electron using light with a wavelength of 313 nm.

First, we convert wavelength to frequency.

$$\nu = \frac{c}{\lambda} = \frac{3.00 \times 10^8\text{ m/s}}{351 \times 10^{-9}\text{ m}} = 8.55 \times 10^{14}\text{ s}^{-1}$$

$$h\nu = \text{W} + \frac{1}{2}m_e u^2$$

$$(6.63 \times 10^{-34}\text{ J}\cdot\text{s})(8.55 \times 10^{14}\text{ s}^{-1}) = \text{W} + \frac{1}{2}(9.1094 \times 10^{-31}\text{ kg})(0\text{ m/s})^2$$

$$\text{W} = 5.67 \times 10^{-19}\text{ J}$$

Next, we convert a wavelength of 313 nm to frequency, and then calculate the velocity of the ejected electron.

$$\nu = \frac{c}{\lambda} = \frac{3.00 \times 10^8\text{ m/s}}{313 \times 10^{-9}\text{ m}} = 9.58 \times 10^{14}\text{ s}^{-1}$$

$$h\nu = \text{W} + \frac{1}{2}m_e u^2$$

$$(6.63 \times 10^{-34}\text{ J}\cdot\text{s})(9.58 \times 10^{14}\text{ s}^{-1}) = (5.67 \times 10^{-19}\text{ J}) + \frac{1}{2}(9.1094 \times 10^{-31}\text{ kg})u^2$$

$$6.82 \times 10^{-20} = (4.5547 \times 10^{-31})u^2$$

$$\boldsymbol{u} = \mathbf{3.87 \times 10^5\ m/s}$$

7.140 **(a)** We note that the maximum solar radiation centers around 500 nm. Thus, over billions of years, organisms have adjusted their development to capture energy at or near this wavelength. The two most notable cases are photosynthesis and vision.

(b) Astronomers record blackbody radiation curves from stars and compare them with those obtained from objects at different temperatures in the laboratory. Because the shape of the curve and the wavelength corresponding to the maximum depend on the temperature of an object, astronomers can reliably determine the temperature at the surface of a star from the closest matching curve and wavelength.